Reject Codes

The following description of reject codes reflects categories of real estate transfers which are typically non-market sales (i.e. rejected for arm's-length). These descriptions are not rigid rules used to always reject sales. The assessor, during the sale verification process, needs to determine whether the specific sale is, in fact, a market or non-market transaction. If the assessor determines the sale is non-market, the most appropriate reject code should be used. If a sale has initially been rejected, but upon further verification is determined to meet reasonable market criteria, the assessor should notify the appropriate Equalization District Office of their change in opinion, along with the basis for that change.

Note: In the expanded explanation section after the chart, the rejection code shown in parenthesis after each reason corresponds to the rejection codes assigned by DOR to identify the reason a sale is not being used in the assessment process. These codes are entered into the automated PAD system by local assessors and DOR in order to identify the reason that a sale submitted through the property transfer return system is not appropriate for use in valuation.

The assessor should select the most appropriate reject code. The assessor should keep in mind that sales are used for two reasons – comparable sales and ratio purposes.

The "9" codes should rarely be used. When they are used, a comment must be made explaining why it was used. Based on information contained in the comments, DOR may change the reject code on the PAD System to more closely reflect the reason for rejecting the sale.

Reject Code Chart

10 Insufficient Market Exposure		
11	Fulfillment of Land Contract	Original land contract was established in a prior year.
12	Gift	Transfer of title made without compensation whether total or partial.
13	To an Exempt Organization or Government (i.e. churches, town, village, city, state, federal)	DOR considers sales from an exempt organization or government to be a market sale.
14	Exempt from Fee	See sec. 77.25 Wis. Stats. for specifics
15	Family, Inheritance, Will, Sales for nominal or no consideration.	May be valid arm's-length, need to investigate.
16	Inter-corporate/Shareholder	Sale is transferring title to another party under the same company umbrella.
17	Convenience, Joint Tenancy or Trust	Sale is transfer of property to change or create title. No consideration involved.

19	Other	Use of this code requires an explanation in the "Comments" area.
20 Insufficient Knowledge of Buyer/Seller		
21	About Real Estate in General	Buyer or seller is un-informed of the Real Estate Market.
22	About the specific Property	Buyer or seller is uninformed of property defects.
29	Knowledge – Other	Use of this code requires an explanation in the "Comments" area.
30 Compulsion		
31	Plottage/Assemblage	Plottage is two or more sites combined to produce greater utility and assemblage is combining two or more parcels usually but not necessarily contiguous, into one ownership or use.
32	Tax, Sheriff or Judicial	Implies transfer of title involuntarily of the owner. Would include foreclosures.
33	To a Relocation Company	Indicates grantor is under duress to sell to move to a new location.
39	Other	Use of this code requires an explanation in the "Comments area.
40 Non-Typical Financing		
41	Exchange	Trading of property, no money involved.
42	Excess Liens	Selling price may include value of liens such as delinquent taxes or special assessments.
49	Other	Use of this code requires an explanation in the "Comments area.
50 Incomplete Bundle of Rights		
51	Correction Deed	Correct error of a prior deed
52	Life Estate	A life estate in the property is retained by someone other than the grantee.
53	Partial Interests	Divided or undivided rights that represent less than the whole parcel.

54	Time Share	Limited ownership interests in, or the rights of use and occupancy of property.
55	Business Value	An enhancement from intangible personal property such as marketing, management skill, trade names, etc.
56	Personal Property	Identifiable portable and tangible objects that are "personal" and not part of the real estate, but are included in the sale.
59	Other	Use of this code requires an explanation in the "Comments" area.
Use these reject codes for market value sales, but not usable for ratio purposes		
71	Split Parcels	There is no separate assessment on the current assessment roll for the parcel that sold.
72	Multiple Districts	Sale has parcels in more than 1 county/municipality.
73	Classes 4, 5, 5m, 6 or 7	Assessment includes lands assessed as classes 4, 5, 5m, 6, or 7
74	Exempt Classes: MFL, PFC	The sale includes lands in Managed Forest Land or Private Forest Crop
75	New construction/remodeling or demolition after Jan. 1	The sale property was changed between the January 1 assessment date and the date of sale.
76	Mixed Classes	See detailed explanation and chart at the end of this chapter for use/reject rules of more than one class of property is assessed.
78	Prior year's sale	A sale that was conveyed prior to the current year.
79	Other	Use of this code requires an explanation in the "Comments" area.

Insufficient Market Exposure (Reject Series 10)

Fulfillment of land contract (Reject code 11)

If the original land contract was established in a prior year, the deed in satisfaction should be rejected. If the original land contract and the deed in satisfaction are occurring in the same year, the original land contract should be used if it is an arm's-length sale in other regards.

Gift (Reject code 12)

A transfer of this type is not a sale at all but a transfer of title made without compensation. It may be a total or partial gift, but if the transfer is made without full consideration, the sale should be rejected.

To an exempt organization or government (Reject code 13)

This type of transfer is more likely to be a reject if the exempt (charitable/non-profit) organization or government body is purchasing the property. Many times the "sale" to such an organization is part gift due to a donated value. Transfers to a government agency may be forced sales, such as condemnation or tax deeds. If the government is acquiring the property for specific purposes the sale price may include other compensation such as damages.

Exempt from fee (Reject code 14)

Transfers that are exempt from the Real Estate Transfer Fee are by definition not arm's-length transactions. The exemptions are found in sec. 77.25, Wis. Stats. Some transfers that are exempt from the payment of fee are more clearly defined in other reject codes on this list (such as correction deeds, foreclosure, etc.). Those types of transactions should be coded to the reject code which more clearly defines the reason for rejection.

Family, inheritance, will (Reject code 15)

Sales between family members must be verified to determine whether they should be rejected or whether they are acceptable as comparables. Sales between some family members for nominal or no consideration are exempt from the transfer fee (reject code 14, sec. 77.25(8) and (8m), Wis. Stats.) and should be identified with this reject code. A family sale should be considered an arm's-length sale if the consideration is consistent with other sales or is based on appraisal value.

Sales in which the estate is the grantor must be verified to determine whether the transfer of title is due to the will/inheritance or if there is duress involved to satisfy the debts of the deceased. If the grantee is an executor or trustee, the sale may not be an arm's-length sale with nominal consideration.

Note that conveyances "By will, descent or survivorship" are exempt from the real estate transfer fee (sec. 77.25(11), Wis. Stats.) and are also exempt from even filing the RETR so the change in ownership may not be readily evident.

Inter-corporate/Shareholder (Reject code 16)

The sale is transferring title to another party under the same company umbrella. Sales to shareholders are not arm's-length sales. Both types of transfers are usually made at prices favorable to the buyer or may indicate a value assigned for accounting purposes.

The presence of corporate names as grantors and grantees does not by itself indicate rejection as an arm's-length sale but indicates that further investigation may be necessary.

Convenience, joint tenancy or trust (Reject code 17)

Transfers of property to simply change the legally titled ownership such as joint tenancy for marital property or trust for estate planning are not actual sales of the property. There is usually an indication on the deed or RETR when a transaction occurs to create joint tenancy. A transfer from an individual to a trust they created may be easy to identify if the trust name includes the names of the individuals. If a trust is the grantee and it is not evident that the grantor(s) are trustees, some investigation is necessary to determine if it is simply a transfer of title or if it is truly an arm's-length sale. If the trust existed prior to the sale, any property purchased by the trustees could be titled to the trust rather than the trustees individually.

Insufficient market exposure – other (Reject code 19)

If a sale is not an arm's-length transaction because of insufficient market exposure for reasons other than those above, it should be rejected.

Insufficient Knowledge of Buyers/Sellers (Reject Series 20)

About real estate in general (Reject code 21)

While it may be rare to find grantor or grantee so ill informed about the value of real estate that the sale is deemed to be a reject, it is a possibility. Verification of the knowledge of the buyer or seller will probably only be discovered by questioning the parties involved and most likely both the buyer and seller need to be questioned in this situation. In the event of a low selling price, the grantee may not admit that the seller was uninformed and the grantee "got a good deal".

About the specific property (Reject code 22)

This situation may arise if there are conditions that affect the value of the sale property whether the conditions are physically located on the property or not. The conditions would not normally be readily evident, for example, contamination, hidden defects, economic factors, etc. For a sale to be rejected as a non-market sale, either the buyer or seller needs to be unaware of the condition and therefore, unaware of its effect on the value of the property.

Insufficient knowledge – other (Reject code 29)

If a sale is not an arm's-length transaction because of an uninformed buyer or seller and that reason is not identified by codes 21 or 22, use reject code 29 and explain the reason for rejection.

Compulsion (Reject Series 30)

Plottage/Assemblage (Reject code 31)

Plottage is the combining of two or more sites under a single ownership in order to develop one site having greater utility and unit value in the aggregate than when each is separately considered. Assemblage is simply the merging of adjacent properties into one common ownership or use.

Tax, sheriff or judicial (Reject code 32)

Tax deeds, sheriffs' sales or judicial sales all imply transfer of title without the consent of the owner. The tax deed referenced here is the transfer of property to the county because of delinquent property taxes. This transfer is exempt from the real estate transfer fee and also exempt from filing the transfer, sec. 77.25(4), Wis. Stats. Counties selling the property may convey the title with a quit claim deed issued to the highest bidder on sealed bids. These re-sales usually indicate the market value of the property.

Sheriff or judicial sales usually result from a foreclosure action by the party who holds the mortgage to the property. If the property was used as security for a debt, foreclosure is the creditor's means of recouping an investment when the financial obligations have not been met. The consideration for this type of sale may have no relationship to the value of the property itself and more likely indicates the financial interest of the mortgager.

To a relocation company (Reject code 33)

A relocation company facilitates the relocation of individuals and families for employment-related moves. The relocation company handles the sale of the employee's home and purchase of a new home. Some companies buy the employees' property directly without the relocation company. These sales should be investigated to determine whether the sale should be rejected.

Compulsion – other (Reject code 39)

If a sale is not an arm's-length transaction due to compulsion by either the buyer or the seller for a reason other than those identified above, it is not an arm's-length transaction.

Non-Typical Financing (Reject Series 40)

Exchange (Reject code 41)

"Exchange" on the RETR may be indicated for various types of property transfers or sales. One could be the simple swap of properties with or without additional consideration. It is important to verify the type of exchange that took place and the total value of the real estate involved. If the consideration shown on the RETR is only the additional consideration above and beyond the value of the property involved in the exchange, the sale should be rejected.

Another type of exchange is the Deferred Like-Kind Exchange described in Section 1031 of the Internal Revenue Code. This exchange is essentially the sale of one investment property and the purchase of another within certain time periods. The transactions must follow the strict rules contained in Section 1031 including the types of property and allowable time frames. The advantage of this "exchange" is that the payment of the capital gains tax due is postponed. Generally, both the sale of the relinquished property and the purchase of the replacement property are separately negotiated transactions. These transactions should not be rejected.

Excess liens (Reject code 42)

A lien is a claim against a property where the property itself is security for payment of the debt. It is an encumbrance on the title. It may be voluntary (with the consent of the owner) as with a mortgage, or involuntary, such as a mechanic's lien.

A lien may entitle the creditor to have the property sold to satisfy the debt. In this

situation the sale should be rejected as a non-arm's-length sale.

Non-typical financing – other (Reject code 49)

Any other factors related to the financing of sale property that render the sale non-arm's-length.

Incomplete Bundle of Rights (Reject Series 50)

There are additional reasons an assessor may choose to reject a transaction even when it doesn't violate the conditions of an arm's-length transactions. For example, correction deeds are not sales and no transfer of property actually occurs. Below are some of the types of transfers that the assessor should not consider in the evaluation of market sales.

Correction deed (Reject code 51)

Correction deeds are used to correct errors in a prior deed. The correction may be in the legal description, names, consideration or other areas of the original document. It is important to check the original sale, if possible, and determine whether the original sale's use or reject status has changed with the corrected information. If the original sale is a current year's sale and has not yet been processed in the PAD (Provide Assessment Data), the corrections should be noted in the Comments section as the sale is processed. If the sale has already been processed, the DOR should be notified of the correction and any change in the assessment and the use/reject status of the sale. The correction deed should be rejected.

Life estate (Reject code 52)

A life estate is defined as the total rights of use, occupancy, and control, limited to the lifetime of a designated party (*The Appraisal of Real Estate*, Eleventh Edition, page 137). This type of partial interest is usually, but certainly not always, indicated on the RETR. A sale that has a life estate retained by a party other than the grantee should be rejected.

Partial interest (Reject code 53)

A partial interest sale is the conveyance of a fractional share of a property by the named grantor. It may be all of the interest of the named grantor such as a one-half interest or a conveyance of some specific ownership interest such as timber, mineral or air rights. This does not include an original land contract which sometimes indicates "land contract interest" because a warranty deed in satisfaction has not yet been filed.

Transferable Development Right (TDR) is a development right that is separated from a landowner's bundle of rights and transferred to another landowner. *(The Appraisal of Real Estate*, Eleventh Edition, page 137). Perhaps the most common current use of TDRs is for preservation of agricultural production or open space**.**

The sale of the TDR itself would be rejected as a partial interest sale. The existence or absence of the TDR on any particular piece of property is not a reason to reject the sale of that property for partial interest.

A *conservation easement* is a voluntary, legally binding agreement that limits certain types of uses or prevents development from taking place on a piece of property now and in the future, while protecting the property's ecological or open-space values. The grantee

of the easement conveyance is typically a conservation group or trust or a government agency. The "sale" of the easement may be a partial or complete gift with income tax benefits for the grantor. The conveyance of the easement itself should be rejected under code 53 (partial interest). Sales of property with existing easements are likely arm's-length transactions and should not be rejected simply because of the existing easement.

Time share (Reject code 54)

Timesharing involves the sale of either limited ownership interests in, or rights to use and occupy residential apartments or hotel rooms. Any time share transfer should be rejected with this reject code.

Business value (Reject code 55)

Business value is sometimes referred to as business enterprise value, going-concern value, blue sky or goodwill. It is a value enhancement that results from items of intangible personal property, such as marketing and management skill, an assembled work force, working capital, trade names, non-realty related contracts or leases, and some operating agreements. In summary, it is the value created by an established operation.

It is important to note that this value may or may not be included in the total value of the real estate listed on the RETR. For the purpose of determining whether the sale is a usable, arm's-length sale for real estate valuation, it is important to separate any non-real estate components of the sale, such as the "business value" which may be included in the sale price.

If this "business value" is included in the consideration on the RETR and cannot be determined and separated from the value of the real estate the sale should be rejected.

Personal property (Reject code 56)

Personal property, as defined by sec. 70.04, Wis. Stats., includes all goods, wares, merchandise, chattels, and effects, of any nature or description, having any real or marketable value, and not included in the term "real property". For the purpose of determining whether the sale is a usable, arm's-length sale for real estate valuation, it is important to separate any non-real estate components of the sale, including any personal property or inventory which may be included in the sale price.

If the value of these items is included in the consideration on the RETR and cannot be determined and separated from the value of the real estate the sale should be rejected.

Incomplete bundle of rights – other (Reject code 59)

If there is a component of the bundle of rights that is not present in the sale and is not addressed in reject codes 51 through 56 the sale should be rejected using code 59.

Assessment/Sale Ratio Rejection Codes (Reject Series 70)

Parcel split (Reject code 71)

A sale that involves a parcel that is not separately described in the assessment roll should be rejected with code 71. This would be a sale of a portion of a parcel described in the assessment roll. The legal description should be reviewed to be sure of a valid comparison rather than relying simply on the parcel number supplied on the Real Estate Transfer.

The assessment should NOT be prorated to use for comparison in the assessment/sales ratio. This would include a condominium sale that does not have a separate land assessment.

Multiple districts (Reject code 72)

If the sale contains property in two or more municipalities, there is not a usable assessment/sales ratio and the sale must be rejected using reject code 72.

Non-market class of 4 or 5 or 5m (Reject code 73)

The law dictates that classes 4, 5 and 5m are not assessed at market value. Therefore, there is no valid assessment/sales ratio of these classes and the sale of properties that contain Class 4, 5 or 5m property should be rejected using reject code 73.

Exempt classes: MFL, PFC, or other exempt (Reject code 74)

A sale that includes any land taxed under the Forest Crop Law (entered into the program prior to January 1, 1986) or Managed Forest Law has no assessment to compare for ratio purposes. Even though a value is listed in the assessment roll the assessment is not valid for ratio purposes.

Any other property that is exempt from general property tax (Federal, State, County or Other) that does not have an assessed value should be reject under code 74 for ratio purposes.

New construction/Remodel after January 1 (Reject code 75)

If the improvements on the sale property have been changed between January 1 and the sale date, the sale should be rejected for ratio purposes. This reject code is for sales which include changes to the improvements after January 1 of the current year. This includes new construction, remodeling, or demolition. Verification of the change in the improvement status is necessary before rejecting a sale for reason 75. Changes to the property prior to January 1 of the year of sale are not a basis for rejecting a sale.

Mixed classes (Reject code 76)

Sales of property with more than one property class are usually rejected. Any sale containing classes 4, 5 and 5m should be rejected (see Reject Code 73.)

There is an exception to this rule. Improved sales totaling less than 20 acres that are a combination of either Classes 1 and 6, or Classes 2 and 6 should not be rejected as a mixed class sale. Please refer to the follow chart for specific examples.

# of Acres	Vac/Imp	Property Classes	Use/Reject
Any	Vacant	All combinations	Reject
0-19	Improved	1 & 6 or 2 & 6	Predominant class must be 1 or 2
0-19	Improved	All other combinations	Reject
20+	Improved	All	Reject

Prior year sale (Reject code 78)

Includes sales that were conveyed prior to the current year.

Other (Reject code 79)

Includes any other reason not identified in codes 71 through 78 that an assessment does not correspond to the property sold in a Market Value sale. Comments must be made when using this reject code. This code should not be used solely because the ratio is out of line with other ratios in the municipality.

Assessment/Sales Ratio

An assessment/sales ratio is the result of two estimates of value – the assessor's and the buyer/seller's. Through the proper interpretation and use of the information found on the RETR the assessor should be able to refine the buyer/seller's estimate of value. This is done by using only valid or arm's-length sales. It should be pointed out that just because a sale fits one of the rejection criteria it is not necessarily totally invalid. Sometimes sales, such as those between relatives, do reflect the market. The assessor must use professional judgment in making this determination.

When sales data has been adjusted (or removed) to reflect market conditions, any error remaining is attributable to the assessment. Evaluating assessment performance is a major goal of assessment sales ratio studies.

Market – Market Imperfections = Assessment Judgment

An assessment/sales ratio can be defined as the percentage derived by dividing the assessed value of a property by the selling price of the same property. The ratio is therefore the result of two estimates of market value, the assessor's and the buyer/seller's.

$$\text{Assessment sales ratio} = \frac{\text{Assessor's estimate of market value}}{\text{Buyer/Seller estimate of market value}}$$

A group of individual assessment/sales ratios forms the basis for an assessment/sales ratio analysis. Take the example of the nine sales (hence nine ratios) for a tax district shown in Figure 10-1. The assessor, in keeping with sec .70.32, Wis. Stats., is attempting to assess at 100 percent of market value. Even if the assessor achieves an overall average of 100 percent, not every ratio will be at 100 percent. In this example, the assessment meets the average of 100 percent however the individual assessments range from 70 percent to 130 percent.

Figure 10-1

Property	Assessments	Sales price	Ratio
1	$250,800	$228,000	110
2	235,000	235,000	100
3	239,400	266,000	90
4	221,000	221,000	100
5	184,450	217,000	85
6	161,350	230,500	70
7	260,000	200,000	130
8	256,450	223,000	115
9	260,000	260,000	100
		Total	900

Average = 100%

Later in this section we will see how the ratio can be refined by using only sales that were not rejected as invalid or compulsive. The DOR has established formal criteria for the rejection of sales.

The assessment process is not an exact science. A specific assessment can be low, average, or high with respect to the average level assessment just as a sale can be low, average, or high within the open market.

Figure 10-2

Case	Assessment	Sale	Ratio
1	Average	Low	High
2	Average	Average	Average
3	Average	High	Low
4	Low	Low	Average
5	Low	Average	Low
6	Low	High	Very low
7	High	Low	Very high
8	High	Average	High
9	High	High	Average

This is a starting point for discussing ratios because "low," "average," or "high" have not been defined in a meaningful way. However, an important message comes through. Ratios are expected to differ (sometimes substantially) from the average. Estimation errors by both the assessor and buyer/seller contribute to these variations.

The crucial question arises: Should low and/or high ratios be discarded as being non-representative? Without further information on the sale, the answer is no. Sales should not be rejected solely on the basis of having a "low" or "high" ratio.

The possible results of two sources of value estimates when each source can have a low, average, or high opinion of value is shown in Figure 10-3. The ratios ranged from 83 percent to 121 percent depending on the assessor's and buyer/seller's estimate of value.

Figure 10-3

Value estimates:	$166,000 - Low
	$220,000 - Average
	$260,150 – High

Case	Assessment	Sale	Ratio
1	$250,800	$228,000	110
2	220,000	220,000	100
3	239,400	266,000	90
4	220,000	220,000	100
5	198,000	220,000	90
6	166,000	200,000	83
7	260,150	215,000	121
8	220,000	200,000	110
9	260,000	260,000	100

Can the sales ratios be directly used to determine the uniformity of assessment? If the sales were perfect indicators of market value, and market value was a single concrete value, the answer would be yes. The problem becomes more difficult because the sale price of an individual property is only an estimate of market value within a range of values from low to high just as assessment is an estimate of value for a specific property within a range of low to high.

To isolate assessment error, the first step is to remove raw sales determined to be poor or questionable estimates of market value. For this review it is crucial that the assessor apply established formal criteria for the rejection of sales.

DOR has established formal criteria for the rejection of sales in its Sales Analysis System (SAS) and Provide Assessment Data (PAD) system. These criteria should be used for ratio development. When the assessor is doing market analysis to establish an assessment for an individual property, all the rejection criteria may not apply. For example, a sale of property located in two municipalities would be rejected from a ratio study because of the separate assessments, but the sale may still be valid when doing comparative market analysis.

The primary principle is that a sale should not be rejected arbitrarily or because the assessor 'feels' it isn't indicative of market value. This amounts to accepting sales which justify (rationalize) the assessor's own beliefs. The worst possible procedure is to reject sales solely on the basis of the ratio. Obviously, any kind of assessment can be made to look uniform if all (or most) sales with "unwanted" ratios are rejected.

If all sales that are not indicative of market value are rejected by the formal criteria then the variations in ratios for the remaining sales are largely due to errors in assessment. The assessment/sales ratios can then be used as direct measures of assessment performance.

Use of Assessment/Sales Ratio Studies

The Wisconsin property tax system operates on two levels: state and municipal. Assessment/sales ratio studies are used at both levels. At the state level the studies are used for equalization of value among local jurisdictions, creating a base for fiscal distribution. The state also uses the results of ratio studies to reveal uniformly assessed values among and within municipalities.

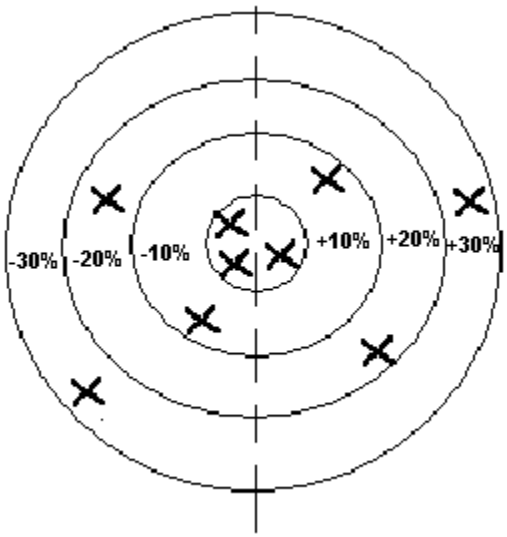
The use of assessment/sales ratio studies can be beneficial to the municipal assessor in a number of ways. The studies can be used as a general appraisal tool to indicate the need for a particular neighborhood, specific class of property, or whole municipality to be reassessed. In conjunction with the reassessment uses, the studies can be used to evaluate mass appraisal methods and budget needs such as manpower and training. Assessment/sale ratios can be used to project the total market value of a specific class of real property or all the real property in a municipality. The use of ratio studies to make annual market equalizations is discussed in depth later in this section. This use, sometimes referred to as trending, helps to maintain uniformity between properties by adjusting the properties not currently reassessed to the level of the most recent appraisals.

A major objective of assessment/sales ratio studies is to determine the degree of assessment uniformity. This measure of assessment performance is gauged by looking at the level of assessment and the degree to which individual assessments differ from that level. This can be more easily understood by thinking of the assessor as a marksman shooting at a target. The bull's-eye represents market value assessments; the concentric rings represent percentages away from the bull's-eye. Each shot represents an individual assessment. The target diagram for Figure 10-1 is shown in Figure 10-4.

NOTE: Assessors should determine the level of assessment annually. The general level of assessment, as determined, should be applied to the personal property. Assessors should not use the level of assessment as determined annually by DOR or the level of assessment indicated on the major class comparison report.

Consider the 9 attempts to hit the bull's-eye in Figure 10-4. The ratios range from 70% to 130% with an average of 100%. If the assessment is uniform, it is expected that most of the ratios would be close to 100%, the average. Although there are extreme ratios, the assessment will be uniform if most of the ratios are near the average. In this example, 7 out of 9 ratios are between 85% and 115% of the bull's-eye; or within ±15% of the bull's-eye. Up to this point, the bull's-eye has only been defined as market value assessments. The assessment would be non-uniform if most of the ratios are far from the bull's-eye.

Figure 10-4



The effect of non-uniform assessments can be seen by looking at properties A and B from Figure 10-5. If both properties are assessed at the sale price ($20,000), then the tax with a mill rate of .034 is $680 for each property owner. The total tax for both properties is $1,360. In this case, one property is assessed at 70% of the sale price and the other property is assessed at 130% of the sale price. The distribution of the tax burden is shown in Figure 10-5.

Figure 10-5

	Sale price	Assessed value	Ratio	Mill rate	Tax
Property A	$20,000	$14,000	70%	.034	$476
Property B	20,000	26,000	130%	.034	884
	$40,000	$40,000	100%	.034	$1,360

The total tax collection for the two properties when one is assessed at 70% and the other at 130% is the same as before, $1,360. The issue is not total tax collection but rather inequity. The over assessed property owner is paying $408 more in taxes for the same priced home as the under assessed property owner. This inequity can only be corrected when each property is assessed in relation to market value (sale price in this example).

Statistical Methods

A series of ratios by itself does not tell much about assessment performance. A basic understanding of statistics is needed to successfully interpret the ratios. Statistics provides a method to understand data by the use of numbers. There are three steps in the studying of statistical data:

1. Collecting
2. Describing
3. Interpreting

This discussion deals with describing and interpreting the data from ratio studies. The following information, provides a basic introduction to statistics. These statistics are meant to aid the assessor in understanding the information available from ratio studies. Additional

information on the topics in this section can be found in any introductory statistics text such as the following: Moore, David S., *Statistics: Concepts and Controversies*, W.H. Freeman and Company, San Francisco, 1979.

Graphic Statistics

Many times the easiest understanding of statistics comes when the numbers are graphically displayed. The first step in displaying the numbers is to make a Frequency Distribution Chart. In making this chart intervals are chosen into which the data can be divided, then the number of occurrences in each interval are recorded and counted. A frequency distribution chart for Figure 10-1 could look like this:

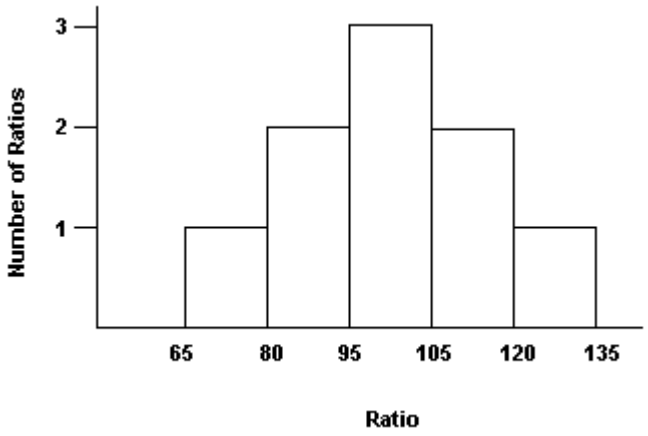
Ratio	Frequency
65 - 80%	1
81 - 95%	2
96 - 105%	3
106 - 120%	2
121 - 135%	1

A frequency distribution chart indicates the most common level of assessment. A symmetrical distribution that included every ratio would indicate that the same number of properties is over assessed as under assessed. The uniformity of assessments can be determined by looking at the degree and nature of the spread near the most common level. If there is a higher concentration of occurrences near the common level then the assessment is more nearly uniform and vice versa. A histogram is another type of graphic representation. Using the same information as in the above frequency distribution chart, the histogram for Figure 10-1 is shown in Figure 10-6.

Two things should be remembered when using a histogram. One, the class intervals on the horizontal axis must be of equal length (e.g., the distance between 65 and 80 must be the same as the distance between 120 and 135). If the distances are not the same, the results will appear distorted. Two, class intervals that do not contain any ratios must not be deleted from the histogram. To do so would make the assessment/sales ratios appear more concentrated than they are. Frequency charts and histograms can help the assessor get an initial feeling for the uniformity of the assessment in the municipality. Assume that the assessor has calculated the following assessment/sales ratios and arranged them from lowest to highest:

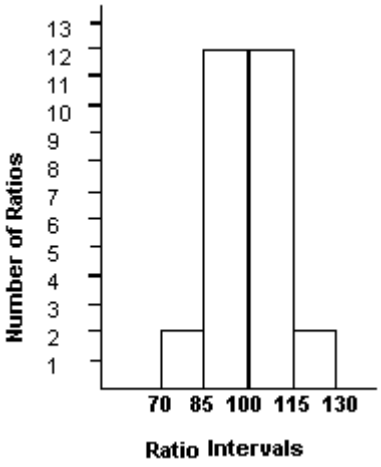
Figure 10-6

78	98	109
85	99	111
87	101	112
88	102	114
89	103	116
91	103	125
92	104	
92	105	
93	106	
95	107	
96		
97		



The assessor then constructs the following frequency chart and histogram:

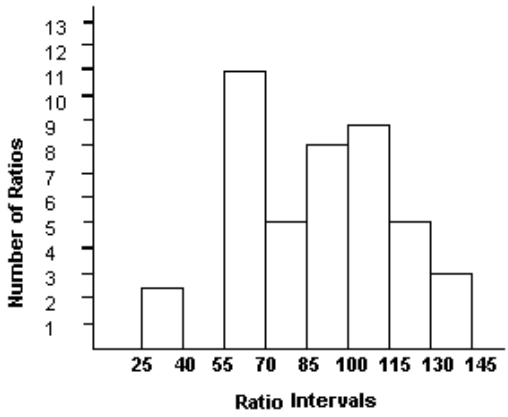
Ratio Interval	**Frequency**
71 - 85%	2
86 -100%	12
101 -115%	12
116 -130%	2



From looking at the frequency chart and the histogram, it can be seen that most of the assessment/sales ratios cluster around 100% and that the assessment is quite uniform.

Contrast that with these assessment/sales ratios from another municipality:

33%	71%	102%	122%
38	75	103	129
56	78	103	132
56	82	105	134
57	84	107	137
58	86	109	
59	87	110	
61	90	111	
63	91	112	
64	93	117	
65	94	119	
66	96	121	
68	98		



The frequency chart and histogram for these ratios is:

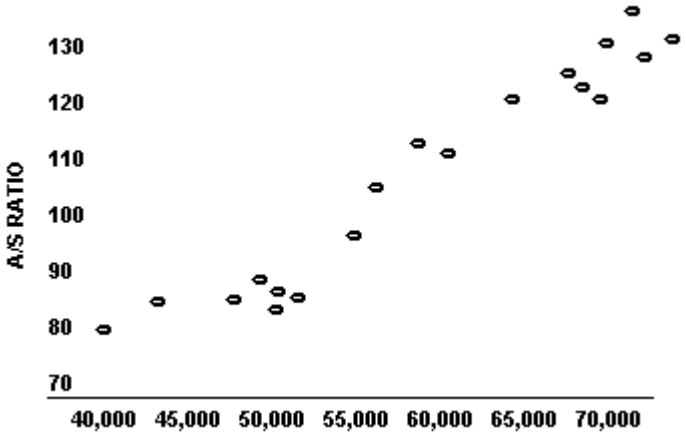
Ratio Interval	Frequency
26 - 40%	2
41 - 55%	0
56 - 70%	11
71 - 85%	5
86 - 100%	8
101 - 115%	9
116 - 130%	5
131 - 145%	3

The assessor can look at the frequency chart and the histogram and see that the ratios do not cluster around 100 percent as they did in the previous example. The wide range of ratios, from 33 to 137 percent indicates a lack of uniformity in the assessments. The large hump in the interval between 55-70 percent could indicate that a particular class or type of property is being assessed at a level other than the general level of assessment.

Another type of graphic representation that the assessor can use to measure uniformity is a scatterplot. A scatterplot is a graph consisting of a dot or point indicating the relationship between a sales price and the assessment for each sale. The scatterplot also gives an indication of inequities in the assessment.

Assume that the assessor has gathered the following information (The scatter plot is to the right):

Sale No.	Assessment Sales Price	Ratio
1	$140,000	80%
2	145,000	82%
3	147,800	83%
4	149,500	85%
5	150,000	75%
6	150,000	82%
7	150,500	80%
8	151,000	90%
9	155,000	95%
10	156,000	100%
11	159,000	107%
12	160,000	105%
13	163,000	110%
14	164,500	120%
15	164,900	119%
16	165,000	117%
17	165,000	123%
18	166,000	125%
19	166,500	118%
20	166,900	120%



By displaying this data on a scatter plot (graph), it is noticeable that lower priced property tends to be under assessed and higher priced property tends to be over assessed. This gives the assessor an indication that there are inequities in the assessment.

While the assessor can get an initial feel for the uniformity of assessment in the municipality through the use of frequency charts, histograms, and other graphic presentations, there is much more information that can be obtained from the assessment/sales ratios. The next section deals with the statistical methods and procedures recommended to further interpret these ratios.

Statistical Testing of Sales Samples

While the assessor can gain a great deal of insight by examining histograms and frequency charts and comparing assessment/sales ratios of various classes and types of properties, there are limits to these analyses. The assessor must remember that an assessment/sales ratio is only a sample of the entire population of the properties in the municipality. While the assessor can, and often must, make assumptions about the entire population based on assessment/sales ratios from a sample, the assessor can never be completely sure that the ratio is representative of the entire population. For the assessor to be completely sure that the assessment/sales ratio truly represents the population, all properties would have to be involved in arm's-length sales. The assessment/sales ratio would then be the ratio of the entire population. Even if 95 percent of the properties sold, the assessor could still not be completely sure that the assessment/sales ratio is truly representative of the entire population since the sale of the remaining five percent could potentially cause some change in the ratio. Of course if 95 percent of the properties sold, the assessor would feel much more confident of the ratio than if only 5 percent of the properties sold.

The assessor is usually working with a small percentage of the total properties in the municipality and this sample is rarely representative of the entire population. Generally, the types of properties that have sold make up a somewhat disproportionate share of the sales sample then they do of the entire population. For example, three bedroom homes may make up 40 percent of the sample, but be only 30 percent of the entire population; one or two neighborhoods may have a great deal of sales activity with the other neighborhoods having little activity; lower value properties may sell more frequently than higher value properties. The assessor can think of many other situations that may affect the ratio. Even though there may be difficulties associated with the use of the assessment/sales ratio from such samples, since ordinary market value is the statutory standard, the assessor would be unwise not to use it to evaluate the assessment level of the municipality and to compare different groups of properties. There are various statistical tests that the assessor can use in conjunction with assessment/sales ratios to aid in making these evaluations. This section explains how the assessor sets up, performs, and uses these tests to evaluate the representativeness of the derived assessment/sales ratio.

Hypothesis Testing

A hypothesis is a supposition tentatively accepted to explain certain facts or to provide a basis for further investigation.

In order to make use of the statistical tests, the assessor states a hypothesis to be tested. For example, the assessor could state the hypothesis that residential and commercial properties are assessed at the same level of assessment.

The assessor would then follow these steps in testing the hypothesis:

Steps in Hypothesis Testing

1. State the Hypothesis. The hypothesis is the statement that the assessor will choose to accept unless the test produces contrary evidence. For this example, the hypothesis will be: "There is *no difference* between the assessment levels of the residential and commercial classes." This is called a *null hypothesis* because it will be accepted unless the test provides contrary evidence.

2. State the Alternative Hypothesis. The alternative hypothesis is simply the opposite of the null hypothesis. In the example (in step 1) the alternative hypothesis is: "The residential and commercial classes of property are assessed at different levels of assessment." The statistical test will provide the evidence to accept one or the other hypothesis.

3. Select the Statistical Test. There are various statistical tests that can be used depending on the type of hypothesis to be tested. This section explains the various tests that can be used.

4. Specify a Confidence Level. As mentioned previously, the assessor can never say with 100 percent confidence that an assessment/sales ratio from a sample is the ratio of the entire population. In the same manner, the assessor can never say with complete certainty that one or the other stated hypothesis is true. To control for this, a desired level of confidence must be selected. The most commonly used confidence levels are 90, 95, 98, and 99 percent. The one most frequently used is the 95 percent confidence level.

5. Perform the Statistical Test. Make the calculation associated with the selected test.

6. Determine from the table whether the calculated number or inferential statistic lies within the desired Confidence Interval. The confidence level selected in step 4 establishes a range. If the calculated number lies within the range, then the assessor cannot reject the null hypothesis. If the calculated number lies outside of this range, then the assessor must reject the null hypothesis and accept the alternative hypothesis. Assume that the assessor selects the 95 percent confidence level and the calculated number lies within the established range. The assessor can then accept the null hypothesis; there is a 95 percent chance that the null hypothesis is true. When the calculated number lies within the acceptable range at the 95 percent confidence level, there is still a 5 percent chance that the assessor has accepted the null hypothesis as true when it is not. As long as the assessor uses a sample of the population, there is always some chance that the sample is not totally representative of the entire population.

Nonparametric Statistics

There are two branches of statistics: parametric and nonparametric. The tests used in this section will utilize nonparametric statistics. Parametric statistics are based on the mean ratio and assume a "normal" distribution. Nonparametric statistics are based on the median ratio. The differences between the mean and median ratio are discussed in the section on Measures of Central Tendency in this chapter. Nonparametric statistics are easy to calculate and understand. They involve ranking, sorting, counting, and relatively straightforward mathematics.

Testing the Level of Assessment

Each municipality is assessed at some level of market value. Sec. 70.32, Wis. Stats., requires that all property be assessed "at the full value which could ordinarily be obtained therefor at private sale" or 100 percent.

The purpose of this statistical test is to determine, at a specified confidence level, whether the calculated assessment/sales ratio from a sample meets the statutory overall level of assessment.

When the sample of sales is 25 or less, the assessor can simply count the number of ratios that are below the desired level of assessment and the number that are above. The assessor can then refer to Table A.

Example:
An assessor attempting to assess property at 100 percent of market value has a sample of 21 ratios; 5 are below 100 percent and 16 are above. The assessor wishes to determine, at the 95 percent confidence level, what the probability is of obtaining this type of a distribution when the desired assessment level is 100 percent. Table A indicates that the probability of obtaining this type of distribution is .013. The .013 indicates that there is only 1.3 percent chance that the distribution is normally distributed around the median. Therefore, the assessor can reject the hypothesis that the desired level of assessment is 100 percent. To not reject the hypothesis at the 95 percent confidence level, the Table would have to yield a probability of .05 or higher. (See note at bottom of table).

Table A

Probabilities Associated with Binomial Test

	x												
N	0	1	2	3	4	5	6	7	8	9	10	11	12
5	.031	.188	.500										
6	.016	.109	.344										
7	.008	.062	.227	.500									
8	.004	.035	.145	.363									
9	.002	.020	.090	.254	.500								
10	.001	.011	.055	.172	.377								
11		.006	.033	.113	.274	.500							
12		.003	.019	.073	.194	.387							
13		.002	.011	.046	.133	.291	.500						
14		.001	.006	.029	.090	.212	.395						
15			.004	.018	.059	.151	.304	.500					
16			.002	.011	.038	.105	.227	.402					
17			.001	.006	.025	.072	.166	.315	.500				
18			.001	.004	.015	.048	.119	.240	.407				
19				.002	.010	.032	.084	.180	.324	.500			
20				.001	.006	.021	.058	.132	.252	.412			
21				.001	.004	.013	.039	.095	.192	.332	.500		
22					.002	.008	.026	.067	.143	.262	.416		
23					.001	.005	.017	.047	.105	.202	.339	.500	
24					.001	.003	.011	.032	.076	.154	.271	.419	
25						.002	.007	.022	.054	.115	.212	.345	.500

Note. – *n* = total number of observations and x = number of observations occurring in the smaller group. Table entries are the probabilities of obtaining a value of *x* as small as or smaller than the indicated value under the assumption that H_0 is true. Probabilities are for a two-tailed test. Probabilities for a one-tailed test are found by multiplying by 0.50, with H_0 accepted whenever one-half or more of the observations do not fall in the direction indicated in H_1.

When the sample size is greater than 25, the assessor should use the following formula:

$$z = \frac{0.5(n-1) - x}{\sqrt{(0.25n)}}$$

n = the total number of ratios in the sample
x = the number of ratios in the smaller group
/ = the square root of the number

Assume that the assessor has 35 ratios, 13 of which are below the target median ratio of 100 percent. The assessor again selects the 95 percent confidence level. The use of the formula gives the following result:

$$z = \frac{0.5(34) - 13}{\sqrt{[(.25)(35)}} = 1.36$$

To interpret this number, the assessor must refer to Table B. In a normal distribution, 50 percent of the ratios lie above the target median ratio and 50 percent below. To allow for this, divide the desired confidence level by 2.

In the example, the assessor would divide .95 by 2 = .475. The assessor would find .475 in Table B and note that this gives a value for z of 1.96. This z value is for half the ratios. To account for all of the ratios, establish a range of +1.96 to -1.96. When the calculated value of z lies within this range, the assessor cannot reject the hypothesis. If the calculated value of z lies outside of this range, then the assessor can reject the hypothesis. In this case, the z value, 1.36, lies within the acceptance range. Therefore, the assessor can, at the 95 percent confidence level, accept the hypothesis that the level of assessment is 100 percent.

NOTE: Observe from Table B that the critical values for the 90 percent confidence level are ± 1.65 and at the 99 percent level are ± 2.58. e.g.: .90 ÷ 2 = .45, .99 ÷ 2 = .495

Table B

Critical Values of *z*

	Second Decimal Place of *z*									
z	00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3143	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note. – Entries in the table give the area under the normal probability curve for positive values of *z*. Areas for negative values of *z* are obtained by symmetry. Thus, for example, the probability of observing $0 \le z < 1.41$ is 0.4207. Similarly, the probability of observing $-1.41 < z \le 0$ is also 0.4207.

Testing for Differences in the Level of Assessment Between Property Groups

One goal of the assessor is to achieve the statutory level of assessment. The principal goal is to assure that there is equity within and between the various classes of property. The assessor can calculate assessment/sales ratios for various classes and types of properties and by looking at the ratios the assessor can get some idea of the equity between the various classes. However, this does not tell the assessor whether the differences between ratios is due simply to sampling chance and is acceptable or whether the difference is due to a difference in the assessment of the various groups. There are two statistical tests that the assessor can use to evaluate the differences between property groups. The *Mann Whitney Test* is used when evaluating *two* groups. The *Kruskal Wallis Test* is used when evaluating *three or more* groups.

Mann-Whitney Test

This test is used when evaluating two property groups. It involves sorting the assessment ratios into two groups, ranking the ratios, and then calculating several straightforward formulas. In this example, the assessor wants to determine whether vacant properties are assessed at a different level of assessments than improved properties. The assessor first states the hypothesis: "Vacant and improved properties are assessed at the same level of assessment." This is the null hypothesis that will be accepted unless the test indicates that it should be rejected. The assessor then calculates and sorts the ratios:

Group 1 Vacant: .517, .528, .531, .539, .548, .549, .555, .574, .581, .588, .594, .600, .608, .613

Group 2 Improved: .495, .503, .524, .529, .536, .542, .550, .556, .561, .569, .573, .577, .584, .595, .597, .603, .610

Next, rank each ratios with 1 being the lowest. Then sum the rankings in each group:

Vacant		**Improved**	
A/S Ratio	**Rank**	**A/S Ratio**	**Rank**
.517	3	.495	1
.528	5	.503	2
.531	7	.524	4
.539	9	.529	6
.548	11	.536	8
.549	12	.542	10
.555	14	.550	13
.574	19	.556	15
.581	21	.561	16
.588	23	.569	17
.594	24	.573	18
.600	27	.577	20
.608	29	.584	22
.613	31	.595	25
	235	.597	26
		.603	28
		.610	30
			261

The assessor can then use either of the following formulas:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \quad \text{or}$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

n_1 = The number of ratios in group 1.
n_2 = The number of ratios in group 2.
R_1 = The sum of the ranks in group 1.
R_2 = The sum of the ranks in group 2.

Using the numbers from the example for the equation.

$$U_1 = (14)\ (17) + \frac{(14)(15)}{2} - 235 = 108$$

$$U_2 = (14)\ (17) + \frac{(17)(18)}{2} - 261 = 130$$

The assessor can now calculate the z value using the following formula:

$$z = \frac{U - n_1\, n_2/2}{\sqrt{[n_1\, n_2(n_1 + n_2 + 1)/12]}}$$

Using U_1, the z value is:

$$z = \frac{108 - (14)(17)/2}{\sqrt{[(14)(17)(14 + 17 + 1)/12]}} = -.437$$

Using U_2, the z value is:

$$z = \frac{130 - (14)(17)/2}{\sqrt{[(14)(17)(14 + 17 + 1)/12]}} = .437$$

Notice the z value is the same regardless of which equation is used. The only difference is the sign. Consult Table B to determine the critical value of z. The assessor uses the 95 percent confidence level, so the critical value of z is ± 1.96. Since our calculated value of z is either + or - .436, and lies within the range of ± 1.96, the assessor cannot reject the null hypothesis: "vacant and improved properties are assessed at the same level of assessment." To use this test either (1) both n_1 and n_2 must contain at least 8 ratios or (2) the larger of the two groups n_2 must contain at least 20 ratios.

This test can also be used to evaluate the levels of assessment between two classes of property. Assume that the assessor wished to determine if there is a significant difference between the levels of assessment for the two classes and had calculated and ranked the following ratios:

Residential		**Commercial**	
A/S Ratio	**Rank**	**A/S Ratio**	**Rank**
.857	16	.756	1
.862	17	.761	2
.864	18	.764	3
.913	24	.768	4
.921	25	.772	5
.926	27	.781	6
.933	28	.785	7
.938	30	.793	8
.945	31	.801	9
.949	32	.804	10
.961	33	.809	11
.968	34	.812	12
.975	35	.817	13
.981	36	.825	14
.984	37	.831	15
.988	38	.865	19
.991	39	.871	20
.997	40	.893	21
	540	.897	22
		.901	23
		.924	26
		.936	29
			280

The assessor can now calculate the value of U, in this case we will use the formula for U_1:

$$U_1 = (18)\ (22) + \frac{(18)(19)}{2} - 540 = 27$$

The value of z would then be:

$$z = \frac{27\ - (18)(22)/2}{\sqrt{[(18)(22)(18 + \ 22 + \ 1)/12]}} = \ .465$$

Assume that the assessor wants to use the 95 percent confidence level with a z value of ± 1.96. Since the calculated z value lies outside the acceptance range, the assessor would reject the hypothesis that the residential and commercial classes are assessed at the same level of assessment.

Kruskal-Wallis Test

This test is used to check the level of assessment between three or more groups. This test is similar to the previous test in that the assessor sorts the individual ratio, ranks them, and then applies a formula.

The assessor wants to determine at the 95 percent confidence level whether residential, commercial, and forest properties are assessed at the same level of assessment. The assessor would state the null hypothesis: "Residential, commercial, and forest properties are assessed at the same percentage of market value." The assessor then sorts and ranks the following 36 ratios:

Residential		**Commercial**		**Forest**	
A/S Ratio	**Rank**	**A/S Ratio**	**Rank**	**A/S Ratio**	**Rank**
.874	3	.892	10	.858	1
.883	6	.898	14	.867	2
.886	7	.901	15	.876	4
.889	9	.914	20	.881	5
.895	11	.919	22	.888	8
.897	13	.924	24	.896	12
.905	17	.930	27	.903	16
.917	21	.933	29	.909	18
.925	25	.939	31	.913	19
.929	26	.950	34	.920	23
.936	30	.956	35	.932	28
.947	33	.959	36	.940	32
	201		297		168

The assessor then uses the following formula:

$$\mathrm{H} = \frac{12}{N\,(N+1)}\left[\frac{(R_1)^2}{N_1} + \frac{(R_2)^2}{N_2} + \frac{(R_3)^2}{N_3}\right] - 3(N+1)$$

NOTE: If there are additional groups, the assessor would add the total rank of that group, square that number, divide it by the number of observations in that group and add it to the groups in the brackets. For example, if there were 5 groups the part of the formula in the brackets would be the following:

$$\frac{(R_1)^2}{N_1} + \frac{(R_2)^2}{N_2} + \frac{(R_3)^2}{N_3} + \frac{(R_4)^2}{N_4} + \frac{(R_5)^2}{N_5}$$

Substituting the numbers in the example gives these results:

$$\mathrm{H} = \frac{12}{(36)\,(37)}\left[\frac{(201)^2}{12} + \frac{(297)^2}{12} + \frac{(168)^2}{12}\right] - 3(37) = 6.74$$

Refer to Table C to determine the critical value for the 95 percent confidence level. The "d.f." on the table represents the "degrees of freedom" of the sample and is the number of property groups minus 1. In this case, the d.f. is 2. Then look across the table from 2 under the 95 percent confidence level to get the critical value of 5.99. Since the calculated value of 6.63 is greater than the critical value of 5.99, the assessor can, at the 95 percent confidence level, reject the hypothesis that residential, commercial, and forest properties are assessed at the same percentage of market value. **NOTE:** There must be at least 5 observations in each property group to use this test.

Table C

Critical Values of Chi Square

	.90	.95	.975	.99	.995	.9995
	Confidence Level for Two-tailed Test					
d.f.	.80	.90	.95	.98	.99	.999
1	1.64	2.71	3.84	5.41	6.64	10.83
2	3.22	4.60	5.99	7.82	9.21	13.82
3	4.64	6.25	7.82	9.84	11.34	16.27
4	5.99	7.78	9.49	11.67	13.28	18.46
5	7.29	9.24	11.07	13.39	15.09	20.52
6	8.56	10.64	12.59	15.03	16.81	22.46
7	9.80	12.02	14.07	16.62	18.48	24.32
8	11.03	13.36	15.51	18.17	20.09	26.12
9	12.24	14.68	16.92	19.68	21.67	27.88
10	13.44	15.99	18.31	21.16	23.21	29.59
11	14.63	17.28	19.68	22.62	24.72	31.26
12	15.81	18.55	21.03	24.05	26.22	32.91
13	16.98	19.81	22.36	25.47	27.69	34.53
14	18.15	21.06	23.68	26.87	29.14	36.12
15	19.31	22.31	25.00	28.26	30.58	37.70
16	20.46	23.54	26.30	29.63	32.00	39.29
17	21.62	24.77	27.59	31.00	33.41	40.75
18	22.76	25.99	28.87	32.35	34.80	42.31
19	23.90	27.20	30.14	33.69	36.19	43.82
20	25.04	28.41	31.41	35.02	37.57	45.32
21	26.17	29.62	32.67	36.34	38.93	46.80
22	27.30	30.81	33.92	37.66	40.29	48.27
23	28.43	32.01	35.17	38.97	41.64	49.73
24	29.55	33.20	36.42	40.27	42.98	51.18
25	30.68	34.38	37.65	41.57	44.31	52.62
26	31.80	35.56	38.88	42.86	45.64	54.05
27	32.91	36.74	40.11	44.14	46.96	55.48
28	34.03	37.92	41.34	45.42	48.28	56.89
29	35.14	39.09	42.56	46.69	49.59	58.30
30	36.25	40.26	43.77	47.96	50.89	59.70

Note.—The region of rejection consists of all values greater than the indicated values.

Measures of Central Tendency

Suppose you were to ask someone to summarize the information in a ratio study by use of a single number. Most people would provide you with an average (a measure of central tendency) of some kind. **CAUTION:** Averages can be misleading. Take the example of the player heights in the two basketball teams below:

Team 1	Team 2
72”	71”
71”	71”
70”	78”
72”	70”
75”	70”
360”	360"
360" ÷ 5 = 72"	360" ÷ 5 = 72"

The average height for both teams is 72", the heights of each player vary by as much as 8".

The three most common measures of central tendency are: the simple mean, the weighted mean, and the median.

The simple mean (average) is computed by adding up the ratios in the sample and dividing by the number of ratios. All sales regardless of dollar amounts are given equal weight. For example, a $10,000 sale counts as much as a $200,000 sale when a simple average is used. The simple mean for Figure 10-7 is: 42 + 48 + 50 + 53 + 58 + 63 + 400 = 714 divided by 7 equals a simple mean of 102%.

Some of the characteristics of the simple mean are the following:
1. It is an easily calculated average using every ratio in the sample.
2. It is easily understood and is the most widely known measure of central tendency.
3. It can be treated algebraically. For example, if two simple means have been calculated for subgroups of the same size, then the overall mean is the simple average of the two-subgroup means.
4. It is sensitive to extreme ratios and thus may not be typical. The simple mean of Figure 10-7 is 102%, which is clearly not a typical ratio in the sample. The extreme ratio of 400% has caused this.

The weighted mean, as the name suggests, is related to the simple mean. The weighted mean when used by the DOR is known as the aggregate ratio. It is calculated by dividing the total of all the individual assessments in a sample by the total of all the individual sale prices in the sample. The weighted mean or aggregate ratio for Figure 10-7 is 277%.

$$\frac{4{,}200 + 9{,}600 + 5{,}000 + 15{,}900 + 11{,}600 + 12{,}600 + 800{,}000}{10{,}000 + 20{,}000 + 10{,}000 + 30{,}000 + 20{,}000 + 20{,}000 + 200{,}000} = \frac{858{,}900}{310{,}000}$$

$$\frac{858{,}900}{310{,}000} = 2.77$$

In calculating the aggregate ratio, large dollar value sales count more heavily than small dollar value sales. For example, a $200,000 sale counts ten times as much as a $20,000 sale. Some characteristics of the weighted mean are the following:
1. It is an easily calculated ratio using every sale in the sample.
2. It is not as easily understood or widely known as the simple mean.
3. It can be treated algebraically.
4. It is sensitive to extreme ratios, thus may not be typical. It can be more sensitive than the simple mean.

The aggregate ratio (weighted mean) is an appropriate measure of central tendency for

estimating the market value of all property (the universe) when given a sufficient sample of sales. It is also appropriate for measuring the relative tax liability of individual taxpayers. This relative tax liability may be computed by dividing the specific assessment/sales ratio by the aggregate ratio. The aggregate ratio measures the level of assessment on a dollar by dollar basis, while the median and simple mean measure on a property by property basis.

It is reasonable that the aggregate ratio of the sample be used to estimate the aggregate ratio of the universe. Therefore, when sales are adequate in number, the DOR uses the sample aggregate ratio to project full market value for equalization purposes.

For the ratios in Figure 10-7 the aggregate ratio is 277% while the simple mean ratio is 102%. The aggregate ratio in this case is even less typical of the sample than is the simple mean. The aggregate ratio is very sensitive to extreme ratios if the extreme sales are large value properties. Conversely, the aggregate ratio is not very sensitive to extreme sales of small dollar value.

The median is quite different from the simple mean and the aggregate ratio (weighted mean). To calculate the median, arrange the ratios in ascending order (from lowest to highest). The median is the ratio located in the middle. If there are an odd number of ratios, the median is an actual ratio. If there are an even number of ratios, the median is the simple average of the two centrally located ratios.

Since there are an odd number of ratios in Figure 10-7, the median is the middle ratio when the ratios are arranged from lowest to highest.

42 48 50 53 58 63 400
↑
Median

If the total number of ratios had been an even number, the median is the average of the two central numbers. For example, the median of the following ratios is 54.

42 48 50 58 63 400
↑
Median

$$\frac{50 + 58}{2} = 54$$

Some of the characteristics of the median are:

1. It is easy to calculate as long as the ratios can be readily arranged in order by size.
2. It is easily understood although not as widely known as the simple mean.
3. It cannot be treated algebraically which limits its use for further statistical calculations.
4. Being an average of position, it is not sensitive to extreme ratios and in this sense tends to be typical. The median is affected by the number of ratios, not by the size of ratios. The extreme ratio of 400% does not distort the median of 53%.

The median is a commonly used measure of central tendency for assessment/sales uniformity studies. To measure uniformity, specific assessment/sales ratios should be compared to an average ratio that is typical of all sales ratios in the sample. As previously discussed, the

median tends to be typical because it is not sensitive to extreme ratios. Uniformity studies are discussed in detail under the heading of Dispersion.

Three measures of central tendency have been defined:

Simple mean of	102%
Weighted mean of	277%
Median of	53%

For the small sample size used in Figure 10-7, the three measures of central tendency that *were used were not very close. When the simple and weighted means are larger than the* median, there are more high ratios than low ratios or as in the example, one extremely high ratio. When the weighted mean is larger than the simple mean, there are large dollar values associated with high ratios. For a large number of sales the three measures will generally be close. Where the ratios are expected to vary, the choice as to the single "best" ratio hinges on what the problem is and how the ratios are to be used. The previously discussed definitions and uses of each measure should aid in the choice. The DOR computes and uses all three.

Figure 10-7

Ratio	42%	48%	50%	53%	58%	63%	400%
Assessment	$4,200	$9,600	$5,000	$15,900	$11,600	$12,600	$800,000
Sale price	$10,000	$20,000	$10,000	$30,000	$20,000	$20,000	$200,000

Simple Mean = 102%
Weighted Mean = 277%
Median = 53%

Dispersion

The major goal of the assessment/sales analysis is to measure assessment performance. The goal of equity dictates that assessments be uniform. Does the measure of central tendency, whether it is the mean or median, indicate the degree of uniformity? Evidently not. A statistical measure of the dispersion (the variation of specific assessment/sales ratios around the average ratio) is needed. Referring back to the target diagram, dispersion measures how far away from the bull's-eye the assessments are. Perhaps the easiest display of variation is the previously discussed frequency chart and histogram. The procedure used by the DOR is outlined and detailed below.

First, arrange the assessment/sales ratios in ascending order; the median is used as the measure of central tendency. If the median ratio is 90%, the objective is to determine whether the individual ratios are generally close to the 90% ratio. Closeness is the degree to which the ratios lie within intervals of ± 15% of the median. Since 90 (the median) x .85 (- 15% of the median) = 76.5 and 90 x 1.15 (+ 15% of the median) = 103.5, ratios to be considered close would have to be within the two intervals:

76.5%	to	90.0%
90.0%	to	103.5%

In this example, assume these intervals contain 5 and 10 sales respectively.
The concept of dispersion can be extended to define further intervals. For example, how many

sales lie in the intervals defined by ± 15% to 30% of the median? Since 90 x 1.30 = 117 and 90 x.70 = 63.0%, the two additional intervals are:

63.0% to 76.5%
103.5% to 117.0%

In the sample, assume that there are 7 and 3 ratios within these intervals respectively.

In this case, all 25 ratios fall within ± 30% so it is not necessary to define further intervals (30% to 45%, etc.).

Summarizing

Interval	Number of sales (Frequency)
63.0% - 76.5%	7
76.5% - 90.0%	5
90.0% - 103.5%	10
103.5%- 117.0%	3
	25

It is useful to graph this frequency table as shown in Figure 10-8. Let the horizontal axis designate the ratio intervals and let the vertical axis designate the number of sales (frequency) within the intervals. Draw a bar to indicate the frequency of sales in each interval. If the assessment is uniform, the bar chart (histogram) will be tall and narrow. If the assessment is not uniform, the bar chart will be short and wide.

To show the contrast between uniform and non-uniform assessment, two hypothetical frequency charts are illustrated in Figures 10-9 and 10-10. The median (90%) and the number of sales (25) is the same for both samples.

For the uniform assessment, Figure 10-9, a unimodal frequency distribution is formed. The chart is unimodal because the two intervals bracketing the median (76.5% to 90.0% and 90.0% to 103.5%) contain a large percentage of the total sales. In simplest terms, a frequency chart is unimodal when a single major "hump" is observed. If the "hump" is in the middle, many properties are being assessed properly.

A more technical reason for the desirability of unimodal frequency charts (centered in the middle) relates to statistical theory. For such statistical determination as that of "adequate" sample size, it is usually assumed that an assessment is "well behaved," that is, the ratios in the universe are normally distributed. (See a basic statistics text for description of the normal curve). The normal curve is bell-shaped and illustrated in Figure 10-11.
Since the normal curve is unimodal, it is encouraging when the frequency charts are found to be similarly shaped. When properties are assessed uniformly, a bell-shaped frequency chart is expected.

The assessment performance shown in Figure 10-10 is undesirable because many assessment sales ratios fall in an interval which does not bracket the median. This indicates that many properties are inequitably assessed.

Figure 10-12 is a skewed frequency chart. A frequency chart is skewed when there is a major hump (concentration of frequency) on one side or the other of the median. This would again indicate that many properties are inequitably assessed. A certain neighborhood or class of property may be dramatically over or under assessed when compared to the other property in the municipality.

In previous frequency charts the intervals have been specified by the end most ratios. From now on the intervals are denoted by percentage distance from the median. The first plus interval is 0 to + 15% while the first negative interval is 0 to -15%, etc.

Figure 10-08

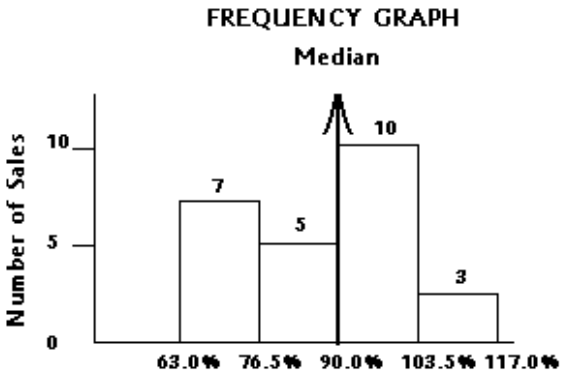


Figure 10-09

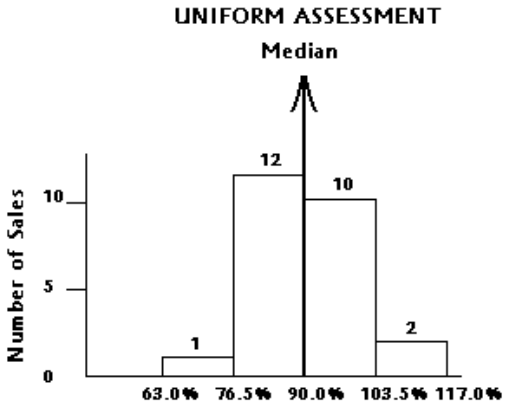


Figure 10-10

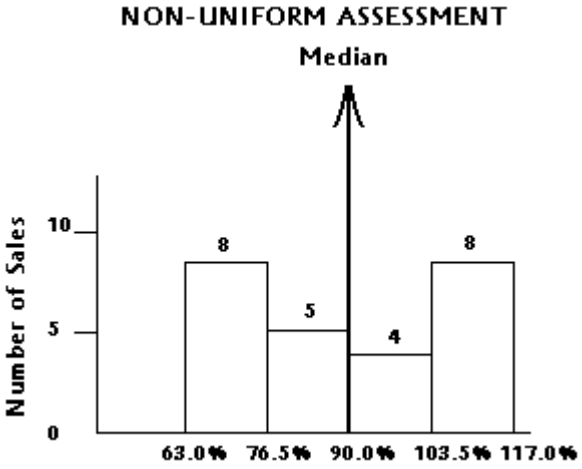


Figure 10-11

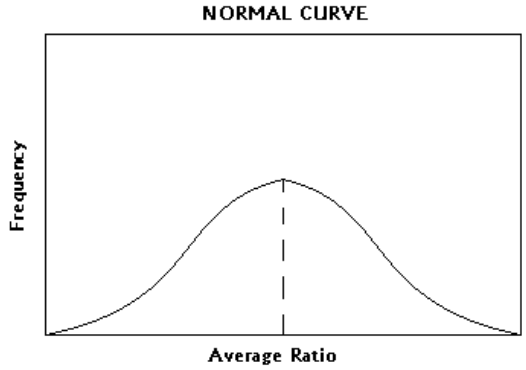


Figure 10-12

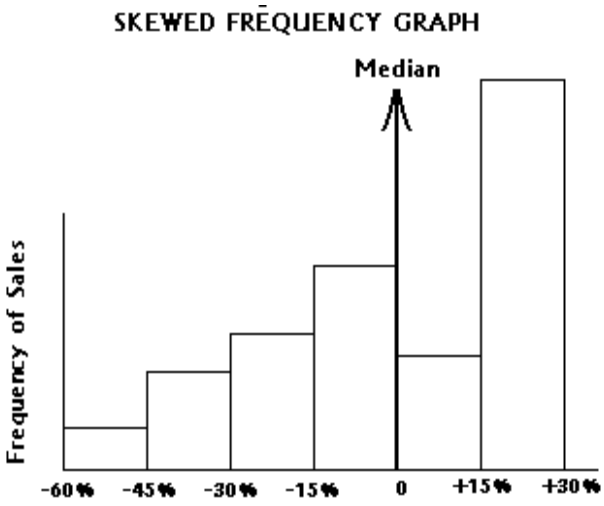


Figure 10-13

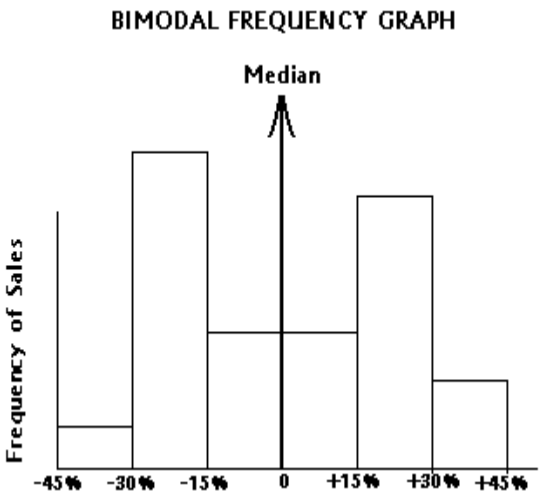


Figure 10-14

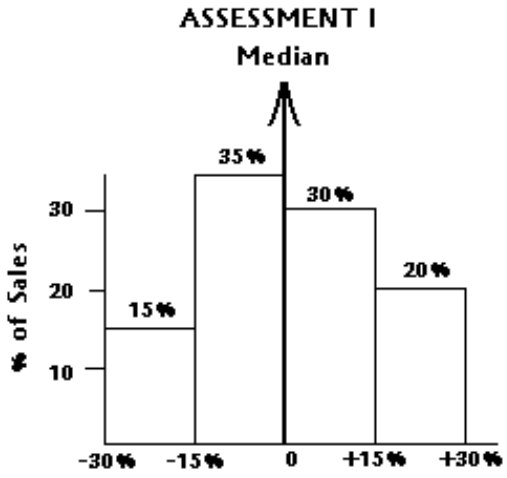
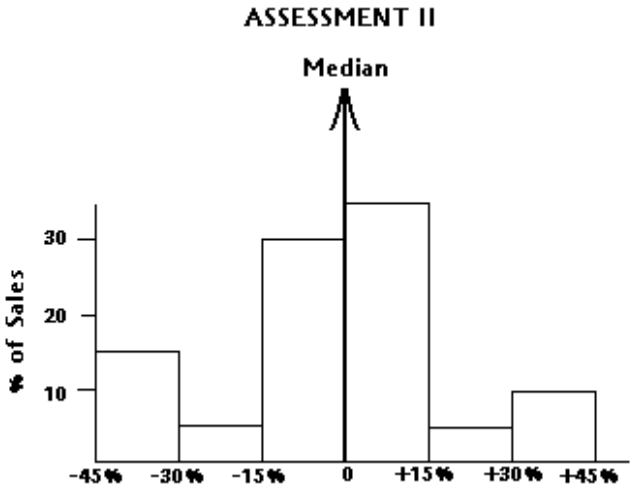


Figure 10-15



A bimodal frequency distribution is illustrated in Figure 10-15. A frequency chart is bimodal when there are two major humps. The term bimodal refers to a measure of central tendency not previously discussed: the mode. The mode is the ratio that occurs most frequently. The mode is not normally used by the DOR since it records only the most frequent ratio, and this ratio may be far from the center of distribution.

Such a bimodal frequency chart indicates poor assessment. This poor assessment may come from several causes. First, the assessor may just be "all over the map" in the assessments. Second, there may be systematic bias in the assessment of different kinds of property. For example, if all classes of property are included in the sales sample, then the tall bar to the left of the median may correspond to commercial property being under assessed. The tall bar to the right of the median may correspond to over assessed residential property.

Alternatively, the two property classes might be reversed. As a further example, suppose the sales sample is solely residential; then perhaps old property is systematically under assessed while new property is systematically over assessed. Third, assessor turnover may have occurred and the two assessors may have vastly different perceptions of market value. Assessor 1 may be responsible for the "hump" to the left whereas Assessor 2 may be responsible for the "hump" to the right.

Such non-uniformity can arise when a local roll is copied from one year to the next and/or when only those properties which have sold are reassessed.

Obviously, much can be learned from the frequency chart, which visually displays the information in a sales sample. The DOR commonly uses the coefficient of concentration and relative coefficient of dispersion to summarize the degree of assessment uniformity to a single number.

The simplest and easiest way to measure uniformity is the coefficient of concentration. This measure is expressed as the percentage of ratios, which lie within ± 15% of the median. Return to the example in Figure 10-10. There were 25 total sales; 5 ratios were within 15% below the median and 10 ratios were within 15% above. The frequency in percentage terms is the following:

5/25	=	20%	within	-	15% of median
10/25	=	40%	within	+	15% of median
		60%	=		coefficient of concentration

Therefore, 60% of the ratios are within ± 15% of the median.

In another example, using the information from Figure 10-1, the coefficient of concentration is 78%, that is, 7 out of 9 of the ratios are within ± 15% of the median.

If the goal is to assess property at no greater than ± 15% from the average assessment level, then the coefficient of concentration tells the extent to which the goal is met.

Note that the coefficient of concentration is related to the discussion on the desirability of unimodal frequency distributions centered around the median. The coefficient of concentration is a single statistic that summarizes the degree to which assessment/sales ratios bracket the median.

The coefficient of concentration does not use all of the information in the frequency chart because it is not concerned with the other intervals (+ 15% to + 30%, + 30% to + 45%, etc.)

To illustrate how the lack of concern for intervals other than -15% to +15% can result in misleading coefficients of concentration, plot two hypothetical frequency charts but change the vertical axis to percentage of sales rather than number of sales and also add another interval on each end. The basic interpretation of the frequency chart remains the same.

The coefficient of concentration for Assessments I and II are both 65%, but it can be seen that Assessment I is superior in overall dispersion. Since the coefficient of concentration loses some dispersion information, one should also examine another measure, the coefficient of dispersion.

The relative coefficient of dispersion measures the average distance (in relative terms) that individual ratios lie from the median. It is calculated by taking each ratio below the median and subtracting it from the median, then taking each ratio above the median and subtracting the median from it. The result is a series of positive differences (deviations). Total these differences and divide by the total number of sales to obtain the absolute coefficient of dispersion. The absolute coefficient is divided by the median to obtain the relative coefficient of dispersion. This calculation for Figure 10-1 with a median of 100% is the following:

100	-	70	=	30
100	-	85	=	15
100	-	90	=	10
100	-	100	=	0
130	-	100	=	30
115	-	100	=	15
110	-	100	=	10
100	-	100	=	0
100	-	100	=	0
Total Deviation			=	110

Absolute coefficient of dispersion is 110/9 = 12.22 Relative coefficient of dispersion is 12.22/100 = .122 x 100 = 12%

The relative coefficient of dispersion is a percentage variable, in this case 12%. Is 12% good or bad? The answer to this question doesn't come easily. It is always possible to make relative comparisons of two assessment performances.

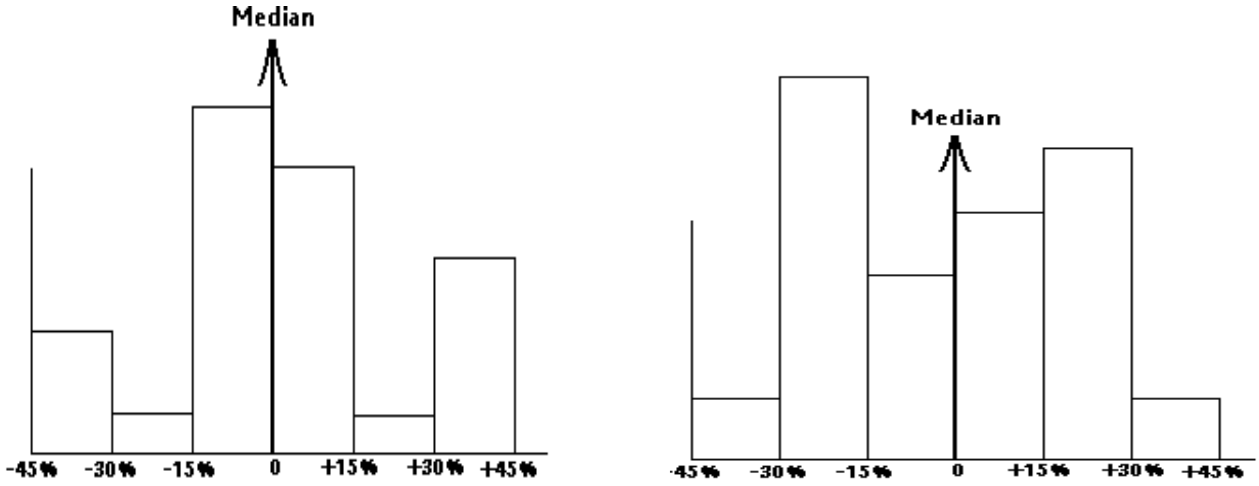
Assessment I (Figure 10-14) can be said to be more uniform than Assessment II (Figure 10-15) if Assessment I has a smaller relative coefficient of dispersion.

But an absolute criterion is desirable. Extensive assessment/sales ratios studies for the State of Wisconsin show that a reasonable degree of uniformity corresponds to a coefficient of dispersion between 10% and 15%. A coefficient of dispersion less than 10% indicates good assessment uniformity.

Although the coefficient of dispersion is a summary of all the variations in the sample, this does not mean that the coefficient of concentration should be ignored. To see this, consider Figures 10-16 and 10-17.

Figure 10-16 Figure 10-17

Assessment III Assessment IV



The coefficient of dispersion can be identical for Assessments III and IV. Even so, one might argue that Assessment III is superior. This follows because Assessment III has more closely met the particular objective of being within ± 15% of the median; this assessor's coefficient of concentration is higher. On the other hand, Assessment IV might be considered better because fewer ratios are far from the median (that is within the third intervals away). The question as to which assessment is superior is, therefore, one of assessment performance goals. To sum up, it is useful to employ both the coefficient of concentration and the relative coefficient of dispersion in conjunction with the frequency chart.

Relative coefficients of dispersion can be compared across tax districts, property classes, or years. They have the tremendous advantage of overcoming the difficulty created by different medians from different universes. Figure 10-18 reflects the DOR's evaluation of coefficients of dispersion.

Figure 10-18

General Property Class	Jurisdiction Size/Profile/Market Activity	COD Range
Residential Improved (single family dwellings, condominiums, manufactured housing, 2-4 family units)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 – 10.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 – 10.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 – 20.0
Income-Producing (commercial, industrial, apartments)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 – 10.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 – 10.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 – 25.0
Residential Vacant Land	Very large jurisdictions/rapid development/active markets	5.0 – 15.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 – 20.0
	Rural or small jurisdictions/little development/depressed markets	5.0 – 25.0
Other Vacant Land (non-agricultural)	Very large jurisdictions/rapid development/active markets	5.0 – 20.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 – 25.0
	Rural or small jurisdictions/little development/depressed markets	5.0 – 30.0

*The COD performance recommendations are based upon representative and adequate sample sizes, with outliers trimmed and a 95% level of confidence.
*Appraisal level recommendation for each type of property shown should be between 0.90 and 1.10.
*PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity. However, PRD standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted.
*Alternatively, assessing officials can rely on the PRB, which is less sensitive to atypical prices and ratios. PRB coefficients should generally fall between -.05 and .05. PRBs that are statistically significant and less than -.10 or greater than 0.10 indicate unacceptable vertical inequities.
*CODs lower than 5.0 may indicate sales chasing or non-representative samples.

Source: IAAO Standard on Ratio Studies

Price Related Differential

Sales data can also be used to indicate the degree to which assessments are regressive or progressive. An assessment is defined to be regressive if low dollar value property is generally over assessed while high dollar value property is generally under assessed. Progressivity is the reverse situation. A useful statistical measure of regressivity/progressivity is the price related differential. The calculation is simple: Divide the simple mean ratio by the aggregate ratio. If the answer is greater than 1, the assessment is regressive. Conversely, an answer below 1 indicates progressive assessment.

Looking at the information in Figure 10-7, the simple mean is 102% and the aggregate ratio is 277%. The price related differential is 102/277 = .37. The result is less than 1 which indicates a progressive assessment. The high dollar values are over assessed and the low dollar values are under assessed. This can be seen by looking at the $200,000 sale that is assessed for $800,000.

The intuition behind this statistic can be developed based on the discussion of central tendency measures. The simple mean counts each sale the same regardless of dollar magnitude. The aggregate ratio places greater weight on sales of large dollar value. If assessments are regressive, the larger value properties are being under assessed. Consequently, the aggregate ratio will be below the simple mean. The price related differential will in turn be greater than 1.

Assumptions

The statistical methods discussed above do not yield accurate conclusions unless some assumptions are met:

1. The sales in the sample are selected on a purely random basis (no bias) from the larger universe of all real property.

   It is known that not all properties in a given class have an equal chance to be selected (to sell). Consider, for example, residential property in a tax district that has a lake. If there is a great demand for recreational property, lakeshore property has a greater chance to sell. A second example is the mercantile class in which some types of real estate such as bank property turn over slowly, if at all. Hence, we are making a strong assumption when we assume random sampling.

2. The universe from which the sample is selected is fairly homogeneous (the properties are similar).

   Homogeneity of the universe (all the real property) refers primarily to the way in which property is assessed. If vacant and improved properties are assessed differently, there are two universes in a class rather than one. In such cases it may become necessary to stratify the sales sample into subsamples that are homogeneous. Though such stratification is ideal in theory, it breaks down in actual practice where we usually have at best a moderate volume of sales. It can be bad practice to stratify to the point that each subsample contains few sales. In the face of a limited number of sales, one is generally forced to make the assumption of a homogeneous universe and proceed.

Summary

The purpose of the procedure outlined in this section is to provide a comprehensive analysis of an assessment ratio series.

The graphical profile provides a universal comparative picture of any assessment/ratio distribution, since the vertical axis is stated in terms of relative deviation from the median.

The coefficient of concentration is an added refinement, which permits one to look more closely at the inner core of the distribution.

It may be said that any distribution, with a higher coefficient of concentration, regardless of the overall coefficient of dispersion is superior to one with a lower coefficient of concentration.

If it is assumed that the median is the best approximation to the common level of assessments in the primary assessment district under analysis, it follows in our example that since 52% of the properties are within ±15% of the median, that 48% of the properties are paying in excess of 15% too much or too little of their fair share of the tax burden.

The purpose of the frequency chart is to provide a meaningful profile of the various assessment ratios, thus permitting a more refined and sensitive analysis of assessment conditions than would a single figure such as the coefficient of dispersion. On the following pages are found four frequency charts (profiles) of various dispersions. The charts are constructed based on a symmetrical distribution of the area under the "normal curve."

Figures 10-19, 10-20, 10-21, and 10-22 represent theoretical symmetrical distributions having a coefficient of dispersion of 10, 15, 25, and 35 respectively. While actual assessment/sales ratios will not, in most cases, present such symmetrical profiles, they will tend to approximate them. Gross variations from the theoretical distributions shown may also indicate further imperfections in either the assessment or the sales sample that was used.

Use of Assessment/Sales Ratios

We have looked at how assessment/sales ratios are developed and how statistical methods can be used to add meaning and understanding to these ratios. Next we will look at how the assessor can use these ratios and statistical methods to achieve better assessment uniformity within the municipality.

Assume that the assessor has calculated the following assessment/sales ratio analysis of the municipality:

Figure 10-19

Class	Total assessed value	Total sales value	Assessment/ sales ratio	Coefficient of Dispersion
Residential	2,450,000	3,000,000	81.7%	21%
Commercial	1,395,000	1,415,000	98.6%	8%
Undeveloped	254,750	249,625	102.1%	7%

If the goal of the assessor is to assess all property at 100% of market value, the assessor would conclude from the analysis that this goal has been met for the Commercial and Swamp/Waste classes; the assessment/sales ratios for the two classes are quite close to 100% and the coefficients of dispersion are excellent. However, the Residential class stands out as not meeting the goal; the assessment/sales ratio is not close to 100% and the coefficient of dispersion is poor. The assessor should further analyze the sales of the Residential class to determine why it is poorly assessed. Assuming that there are an adequate number of sales, the assessor should stratify the sales by neighborhood, by style, by age, and other features to determine whether just one or two types or locations of residential property are under assessed or if it is the entire class that is under assessed. Care must be taken to assure that each substrata contains more than just a few sales. The assessor may find it necessary to use prior years' sales in order to have an adequate number of sales for analysis. The prior years' sales would have to be adjusted for time as required.

Assume that the assessor calculates the following assessment/sales ratios for the various residential neighborhoods:

Figure 10-20

Neighborhood	Assessed Value	Sales value	Ratio	Dispersion
A	405,000	400,000	101.3%	8%
B	445,000	450,000	98.9%	3%
C	245,000	475,000	51.6%	26%
D	515,000	525,000	98.1%	7%
E	540,000	550,000	98.2%	4%
F	300,000	600,000	50.0%	32%

From looking at the various ratios, the assessor could conclude that neighborhoods A, B, D, and E are quite close to 100% and the coefficient of dispersion is excellent. However, neighborhoods C and F are both under assessed and have poor coefficients of dispersion. The assessor can now concentrate on revaluing these two neighborhoods to bring them and thus the entire Residential class up to 100%.

The assessor could also stratify the sales by style, age, or other features to determine if there is any particular type of property that is poorly assessed. Again, care must be taken to assure that there is an adequate number of sales for each substrata for meaningful analysis. The more substrata that the assessor can identify and analyze, the easier it will be for the assessor to correct assessment problems. If, in our previous example, the assessor can also stratify the sales within the neighborhoods by style, age, and other features, the assessor may further narrow the properties that need attention. For example, if analysis of neighborhood C shows that all property meets the market value goal except for property built within the last two years, the assessor's efforts can be concentrated on that substrata. Efforts by the assessor to define and analyze various substrata can focus the attention of the assessor on those substrata that are in need of revaluation and prevent the assessor from spending time and effort on those areas that already meet the criteria of market value assessment.

The assessor can also use the assessment/sales ratio analysis to show the need for a

revaluation of the entire municipality. Assume the assessor calculates the following assessment/sales ratios for the municipality:

Figure 10-21

Class	Total Assessed Value	Total Sales Value	Assessment/ Sales Ratio	Coefficient of Dispersion
Residential	2,987,000	4,754,000	62.87%	23%
Commercial	1,348,000	1,500,000	89.9%	18%
Undeveloped	322,000	655,000	49.2%	25%

The assessment/sales ratios are far apart and the coefficient of dispersion is fair to poor. Unlike the previous example, the assessor cannot concentrate on just one class or type of property. All classes and types of property will have to be reviewed by the assessor. A complete revaluation of the municipality may be the best way to provide the necessary resources to complete this overall review in a timely manner.

In a similar manner, the DOR uses assessment/sales ratios to ensure equity between municipalities. Assume that County "K" has only three assessment districts: Town "T," Village "V," and City "C." County "K" wishes to levy a property tax in the amount of $40,000. Since the county has no assessment roll of its own, it will allocate or apportion the total levy among the three districts. The following chart shows the county tax being apportioned based on the assessed values of the municipalities:

Figure 10-22

	Assessed Value	% of Total County Assessed Value	County Tax Levy	Municipal Portion of County Levy
Town "T"	$ 800,000	19.5% x	$40,000	= 7,800
Village "V"	300,000	7.3% x	$40,000	= 2,920
City "C"	3,000,000	73.2% x	$40,000	= 29,280
County "K"	$4,100,000	100%		$40,000

If all three municipalities are assessing property at 100% of market value, then this is a fair allocation of the county levy and equity is achieved. However, not all municipalities assess at full market value. Assume that the DOR through analysis of the sales in the three municipalities has calculated the following assessment/sales ratios: Town "T": 40%, Village "V":30%, City "C":60%. The full or equalized value for each of the municipalities can be determined by dividing the assessed value by the assessment/sales ratio:

Figure 10-23

	Assessed Value	Assessment/Sales Ratio	Full or Equalized Value
Town "T"	800,000	40%	$2,000,000
Village "V"	300,000	30%	1,000,000
City "C"	3,000,000	60%	5,000,000
County "K"	$4,100,000		$8,000,000

The county levy can then be calculated based on the full or equalized value:

Figure 10-24

	Equalized Value	% of Total County Equalized Value		County Tax Levy		Municipal Portion of County Levy
Town "T"	$2,000,000	25%	X	$40,000	=	$10,000
Village "V"	1,000,000	12.5%	X	$40,000	=	5,000
City "C"	5,000,000	62.5%	X	$40,000	=	25,000
County "K"	$8,000,000	100%				$40,000

It can be seen that by using the assessed values to apportion the county levy, the municipality that assesses at a lower level of assessment pays a smaller share of the county levy and, conversely, the municipality that assesses at a higher level of assessment pays a higher share of the county levy. However, by using the full or equalized value, each municipality bears its fair share of the county levy. The apportionment of school tax, sanitary districts, and other apportionments would be done in a similar manner.

Annual Assessment Requirement

Assessments should be set annually in order that property tax burdens may be distributed equitably. This annual assessment requirement implies a conscious reevaluation of all appraisal factors used and, when one or more factors have changed, a recalculation of the assessment.

Actual View

Assessors need to follow state law, sec. 70.32, Wis. Stats., and develop assessments at full value based upon actual view of the property or the best information available. An interior and exterior view provides the most accurate information for developing assessments. However, an interior and/or exterior view may not always be possible. If a written request for an interior and/or exterior view is refused (see Chapter 5-10 Notification Process), the assessor generally should not enter the property. The assessor should base the assessment on the best information available. The following explains the process to collect information and the best sources of information.

Proceed with the standard assessment discovery, listing and valuation processes as described by state law and the Wisconsin Property Assessment Manual. The following lists the sources of information the assessor can consider with the best sources listed first:

1. Request a view of the property (see Chapter 5-10 Notification Process)
2. View the property from a public area such as a road
3. Request data from the property owner, (e.g., construction contracts, leases, operating expenses, receipts, blueprints, video and/or photographs of the improvements, etc.)
4. Obtain other information, (e.g., sales listing information and building permits)

If these sources of information do not allow the assessor to develop a value, an interior view is required. As an example, if the property has no prior improvement inspection, there is no view of the property from a public area and the property owner has provided no information. With this type of unique situation, the assessor may request a special inspection warrant under sec. 66.0119, Wis. Stats. This option should be used only when necessary.

Obtaining a special inspection warrant requires three forms:

- An affidavit detailing the facts giving rise to the need for a warrant
- The special inspection warrant itself. The warrant will also advise the homeowner of the lawful basis for the inspection of his home and describe the search's proper limits including identification of the assessor as one with the authority to search.
- Return of Officer

The completed affidavit and warrant should be brought to a local magistrate. Contact the local clerk of courts to determine hours when a magistrate is available. The local magistrate will determine whether or not facts exist to support the issuance of the warrant. If so, the warrant will be signed by the magistrate. The assessor and peace officer or sheriff may then execute the search. After completion of the search, the official paper work (endorsement on warrant and return of officer) should be completed and filed by the assessor. Please see the Appendix for sample special inspection warrant documents.

Trending Factors

The IAAO *Standard on Mass Appraisal of Real Property*, April 2013, defines trending as adjusting the values of a variable for the effects of time. Usually used to refer to adjustments of assessments intended to reflect the effects of inflation and deflation and sometimes also, but not necessarily, the effect of changes in the demand for micro-locational goods and services. A trending factor is defined as a figure representing the increase in cost or sale price over a period of time. Wisconsin case law holds that the application of an across-the-board percentage factor to all property of a class in a county does not satisfy the annual reassessment requirement. (See: *State ex rel. Kaskin v. Bd. of Review of Kenosha Co.*, 91 Wis. 2d 272, 282 N.W.2d 620 (Ct. App. 1979)). The annual reassessment requirement does not demand that all properties must be revisited or that an on-site re-viewing be performed annually, although the more frequent the re-viewings the better. For manufacturing property a five-year cycle is required; for counties under a county assessor system, a four-year cycle is mandated.

The application of assessment trending factors has been accepted by the court as long as different factors are applied to different subsets of properties and encompass the same factors that were considered in establishing the initial assessment. Use of comparable sales requires more than determining arm's length transactions in an entire class throughout a county, such other factors as location, improvements, size or use, and date of sale are appropriate to consider when evaluating comparable sales. (See: *Rosen v. City of Milwaukee*, 72 Wis. 2d 653, 242 N.W.2d 681 (1976); *State ex rel. Kaskin v. Bd. of Review of Kenosha Co.*, 91 Wis. 2d 272, 282 N.W.2d 620 (Ct. App. 1979)).

Glossary

Coefficient of Concentration: Percentage of ratios which lie within ± 15% of the median; measures assessment uniformity.

Coefficient of Dispersion (relative): Take each ratio below the median and subtract it from the median, then take each ratio above the median and subtract the median from it. Sum the differences and divide by the total number of ratios. Then divide this result by the median; measures the average distance (in relative terms) that individual ratios lie from median.

Coefficient of Variation: Standard deviation divided by the mean times 100; indicates the degree of concentration or spread in the distribution of assessment ratios.

(95%) Confidence Interval: 1.96 times the standard error of the mean; establishes interval in which the assessor can be 95% confident that population mean ratio will be included.

Mean, simple: Add the ratios in the sample and divide by the number of ratios; measure of central tendency (average).

Mean, weighted (aggregate ratio): The total of all individual assessments divided by the total of all individual sales; measure of central tendency (average).

Median: Arrange ratios in ascending order; if there are an odd number of ratios the median is the ratio located in the middle, if there are an even number of ratios the median is the average of the two; a measure of central tendency for uniformity.

Mode: Ratio that occurs most frequently.

Price related differential: The simple mean divided by the aggregate ratio; indicates the degree that assessments are regressive (if greater than 1) or progressive (if less than 1).

Standard deviation: The square root of the variance; measures dispersion and variability of normally distributed data.

Variance: Take the difference of each ratio from the mean, square each of the differences and total the squares, then divide the sum by the number of ratios (n); needed to arrive at the standard deviation and to measure spread or variability. In some situations, n-1 is used as the divisor to provide a more unbiased estimator of the population variance.

Sources

Almy, Richard, and Gloudemans, Robert (2011). *Fundamentals of Mass Appraisal*. Kansas City, Missouri: International Association of Assessing Officers.

Elzey, Freeman F. *A First Reader in Statistics*, Second Edition. Monterey, California: Brooks/Cole Publishing Company, 1974.

International Association of Assessing Officers: Technical Standards Committee (2013). *Glossary for Property Appraisal and Assessment*. Kansas City, Missouri: International Association of Assessing Officers.

International Association of Assessing Officers (2010). *Property Assessment Valuation*. Kansas City, Missouri: International Association of Assessing Officers.

Moore, David S. *Statistics, Concepts and Controversies*. San Francisco, California: W. H. Freeman and Company, 1979.